Reconstruction of neural activity using kinetic Ising model

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Introduction

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What's the goal?

- Reconstruction of the neuron response given a periodic visual stimulus
- Scalability of the project for thousand of neurons

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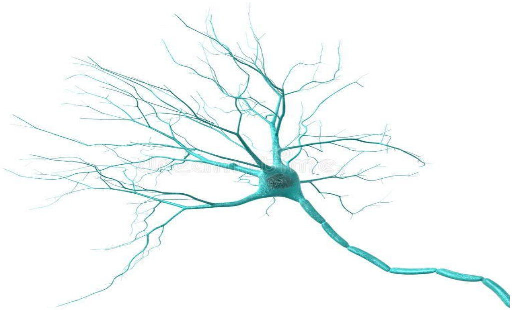
Is it possible to replicate the neurons signals and categorize their importance?

Zebra fish



Introduction

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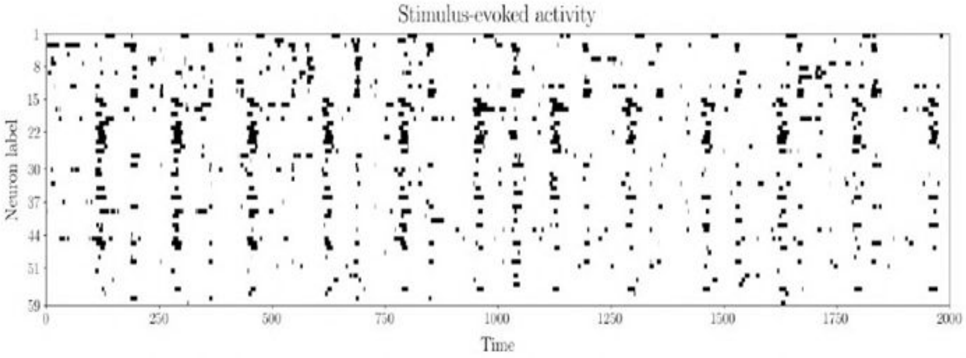
Data?

- Real neurons activity over time

- Synthetic generated data



Probability of flipping

$$\boxed{p_{flip}} = \gamma \delta t \frac{1}{2}[1 - s_i(t) tanh H_i(t)]$$

Ising Model

$$H_i(t) = h_i + \sum_{j \neq i} J_{ij} s_j(t)$$

External field

Coupling matrix

To reconstruct the model we need to discretize data

Theoretical Foundations

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Theoretical base

Infer the parameters of the (asynchronous) Ising model

$$L = \sum_i \frac{1}{N_{\tau_i}} \sum_{\tau_i} [\, s_i(\tau_i + \delta t)\theta_i(\tau_i) - logcosh\theta_i(\tau_i)]$$

- Spin history ✔
- Updated times ✘

Pseudo-Likelihood

(Unconnected) covariance matrix

$$\begin{cases} C_{ij} = \langle s_i(t) s_j(t) \rangle_t \\ \dot{C}_{ij} = \delta t^{-1} [\langle s_i(t+\delta t) s_j(t) \rangle_t - C_{ij}] \\ L = \sum_i \frac{1}{N_t} \sum_t [h_i s_i(t) + \sum_{i \neq j} J_{ij} \left(\frac{\dot{C}_{ij}}{\gamma} + C_{ij} \right) - logcoshH_i(t)] \end{cases}$$

ref. [2]

Theoretical Foundations

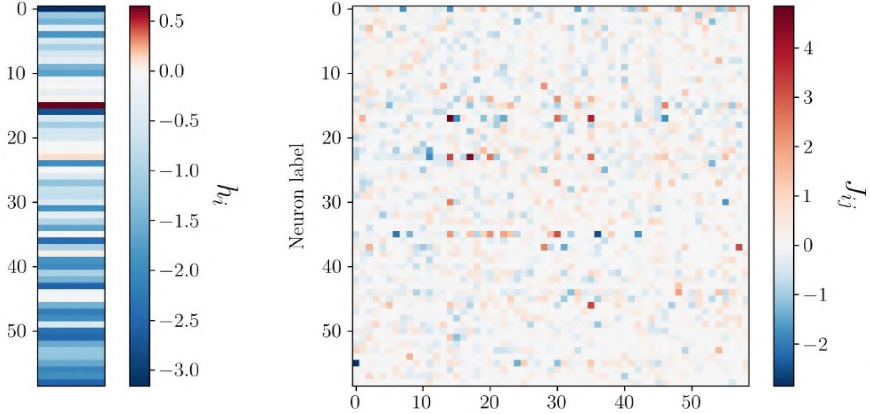
Given the Pseudo-likelihood → ∇L

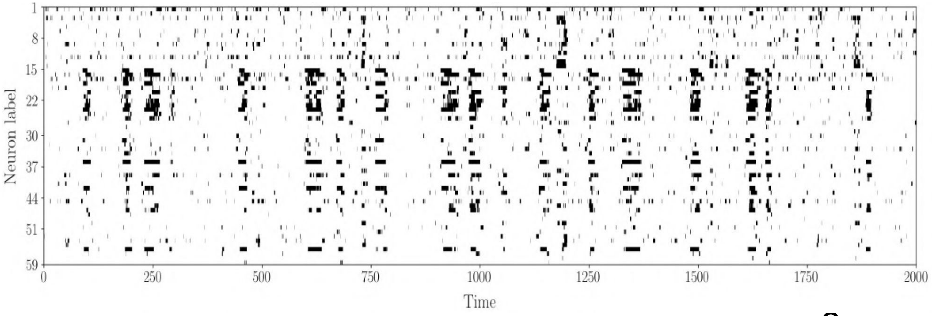
$$\begin{cases} \dfrac{\partial L}{\partial h_i} = \langle s_i(t) - tanh H_i(t) \rangle_t \\ \dfrac{\partial L}{\partial J_{ij}} = \gamma^{-1} \dot{C}_{ij} + C_{ij} - \langle [tanh H_i(t)] s_i(t) \rangle_t \end{cases}$$

Algorithms implemented for maximization:

- GA (mom)
- ADAM
- RMS prop
- NAG

Detailed results later...





Simulation

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$$p_{flip} = \gamma \delta t \frac{1}{2}[1 - s_i(t) \tanh H_i(t)]$$

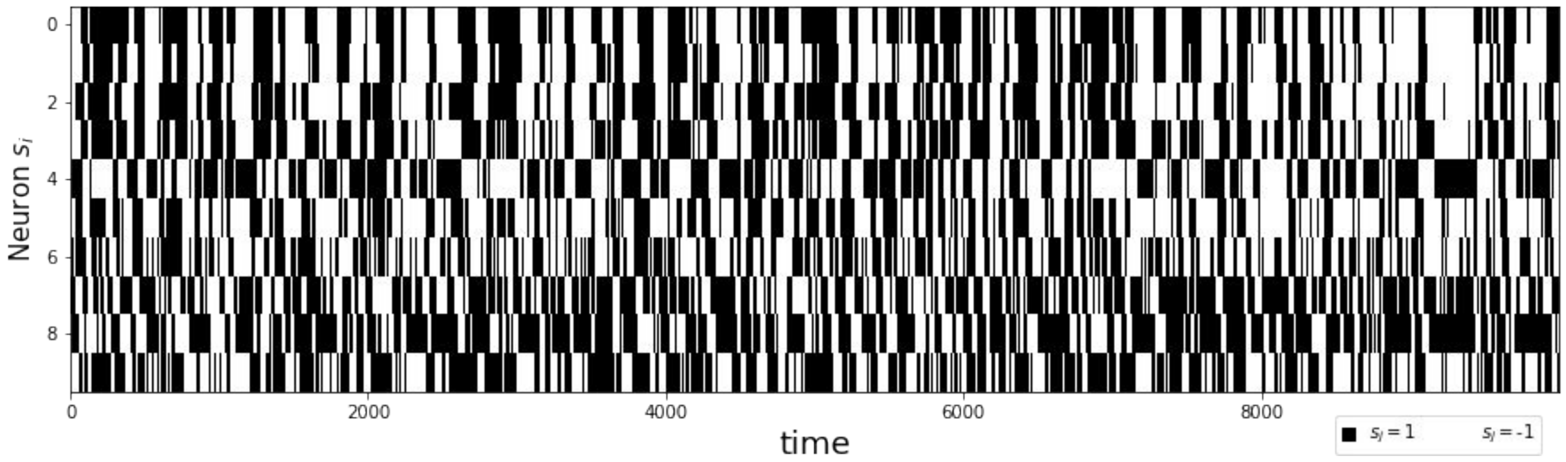
how to infer real-data parameters?

SIMULATION

- ➔ few neurons
- ➔ 100/1000 neurons

```
def simulation(J,h,gamma,steps,dt):

    S = [-np.ones(N)]

    for i in tqdm(range(1,steps)):
        s = np.copy(S[-1])
        H = h + np.dot(J,s)
        p_flip = gamma*dt*0.5*(1-s*np.tanh(H))
        p = np.random.rand(N)
        s[p<p_flip]*=-1
        S.append(s)

    S = np.array(S).T

    return S
```

10 neurons: simulation




```python
J = np.random.normal(0,1,(N,N))
np.fill_diagonal(J,0)
h = np.random.normal(0,1,N)
```

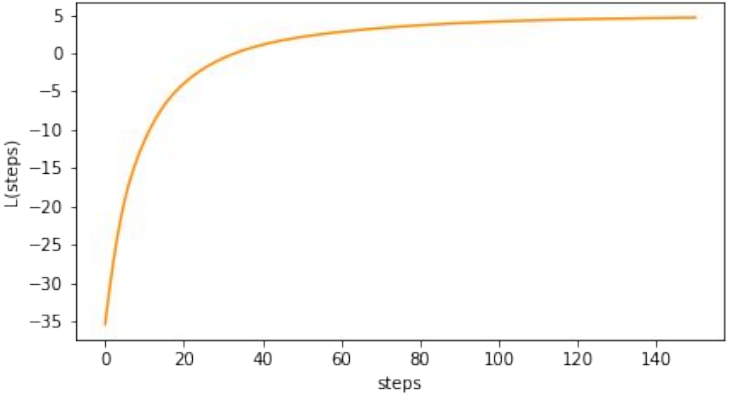
δt = 0.1

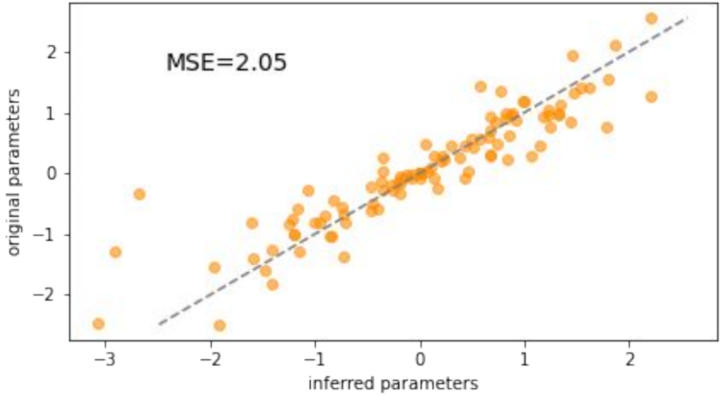
10^6 time steps

gamma = 1

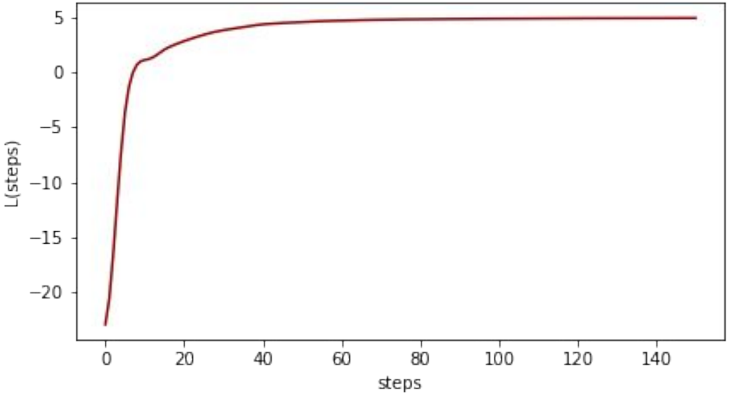
10 neurons: inference

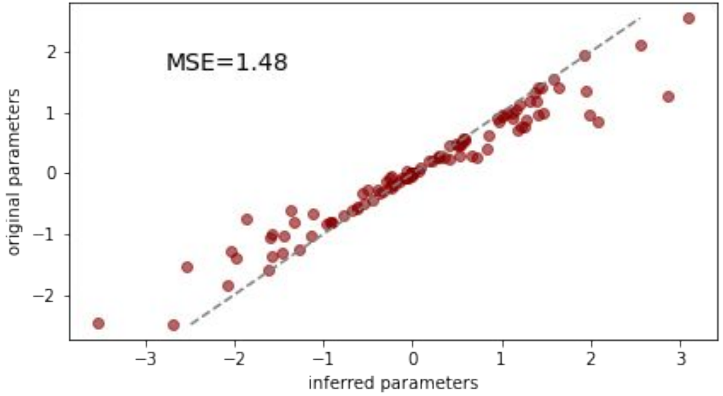
Gradient Ascend



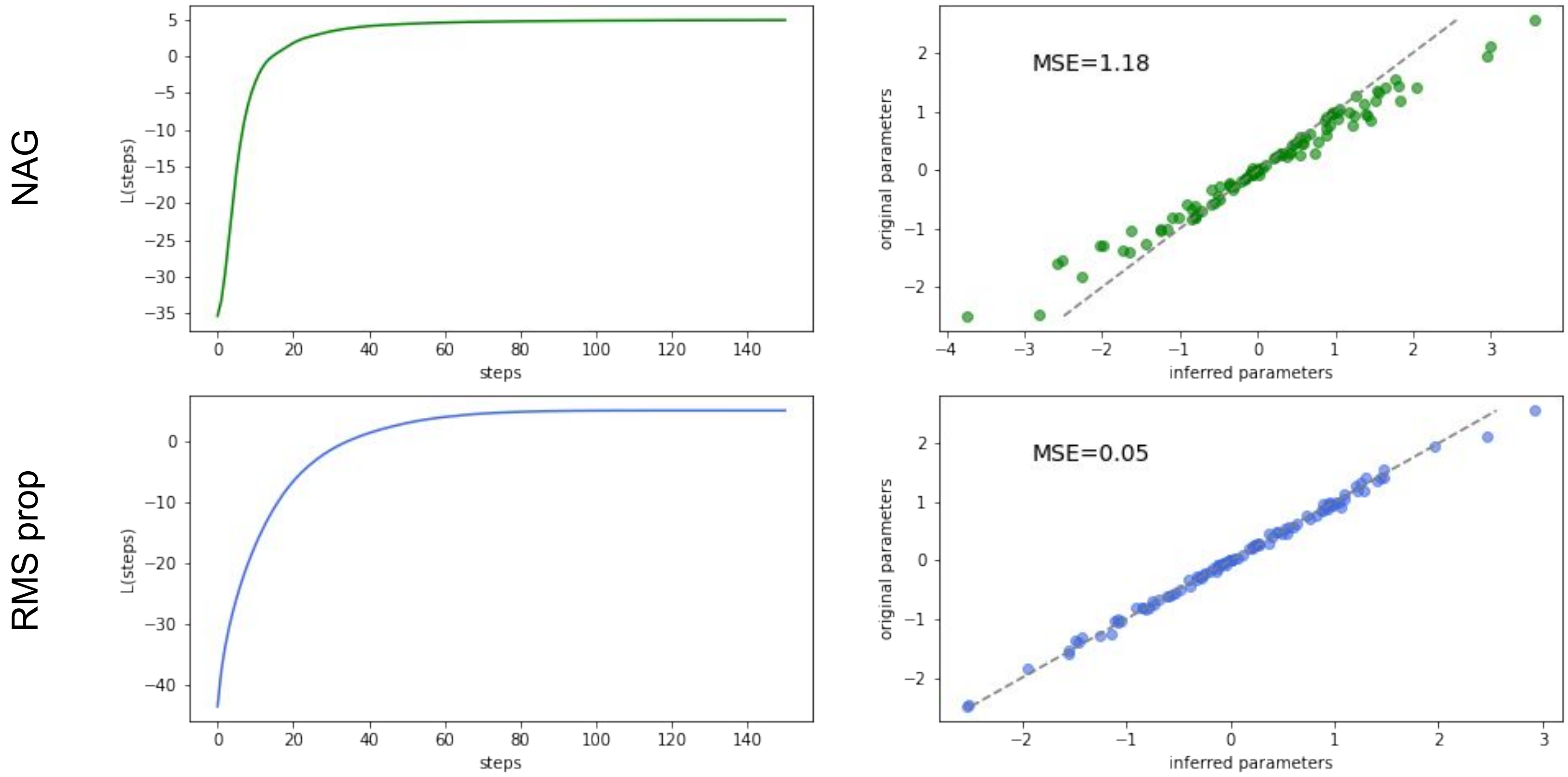


Gradient Ascend with momentum





10 neurons: inference



10 neurons: inference

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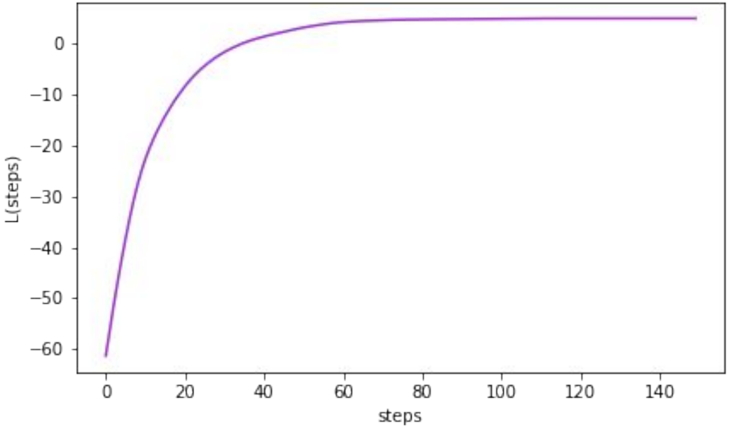
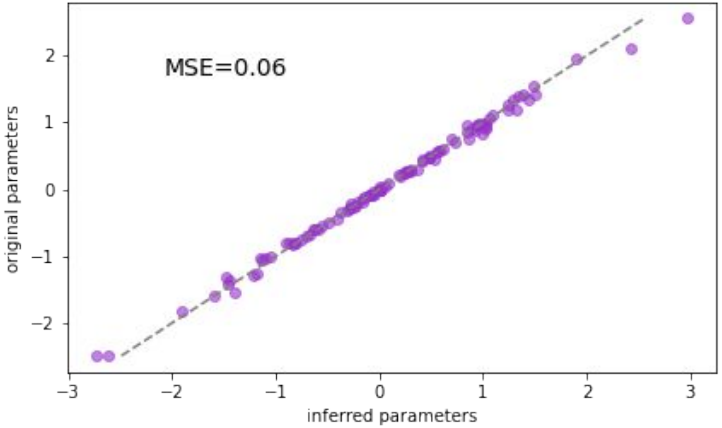
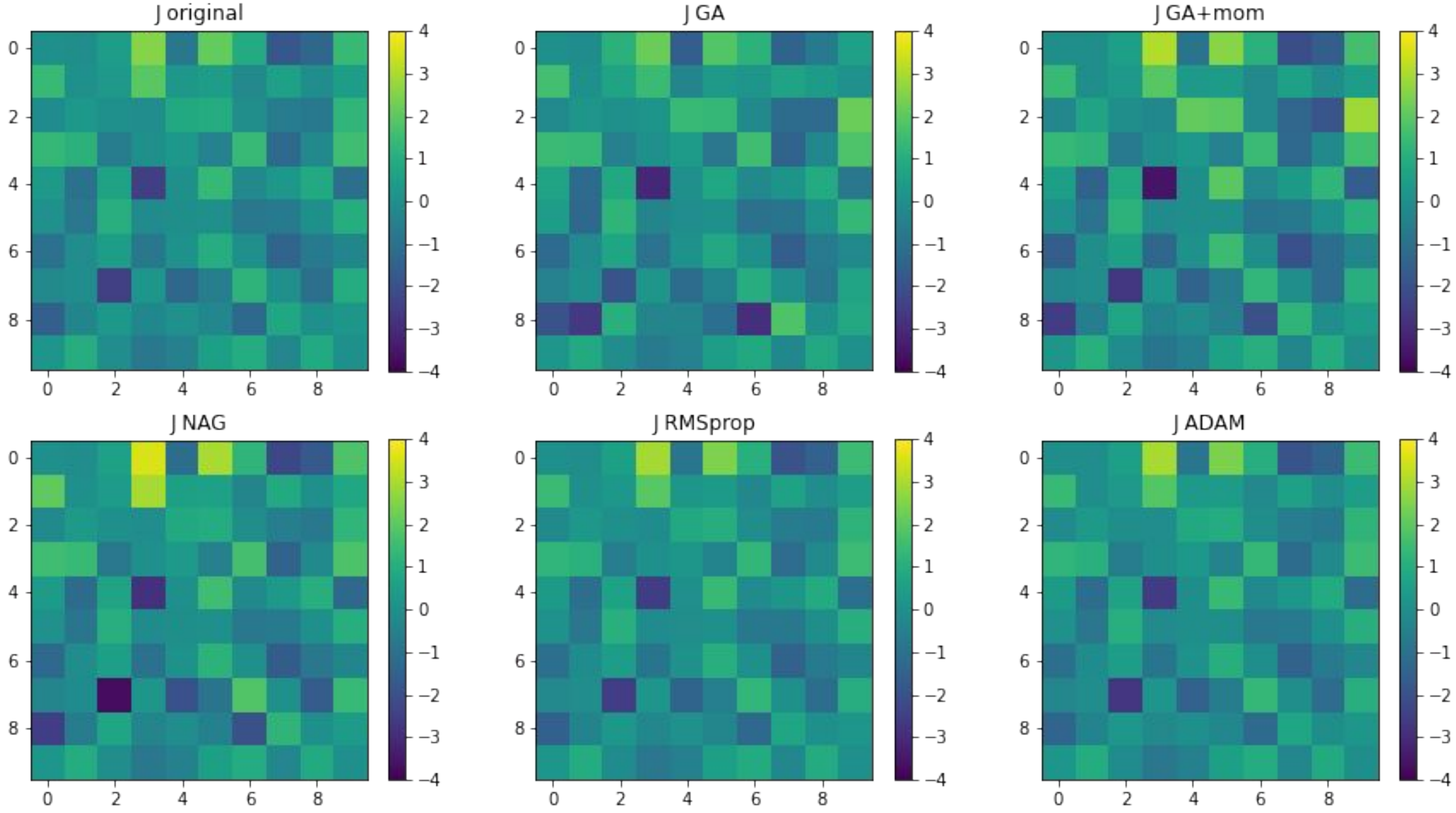
ADAM

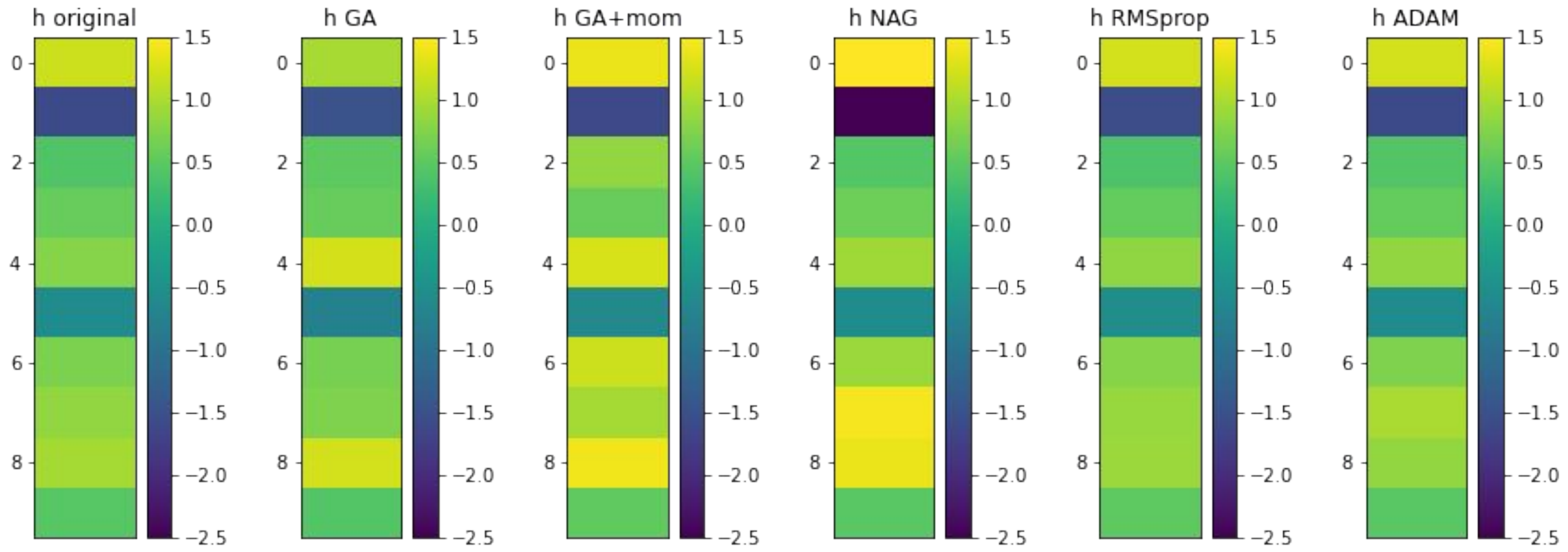




10 neurons: couplings J

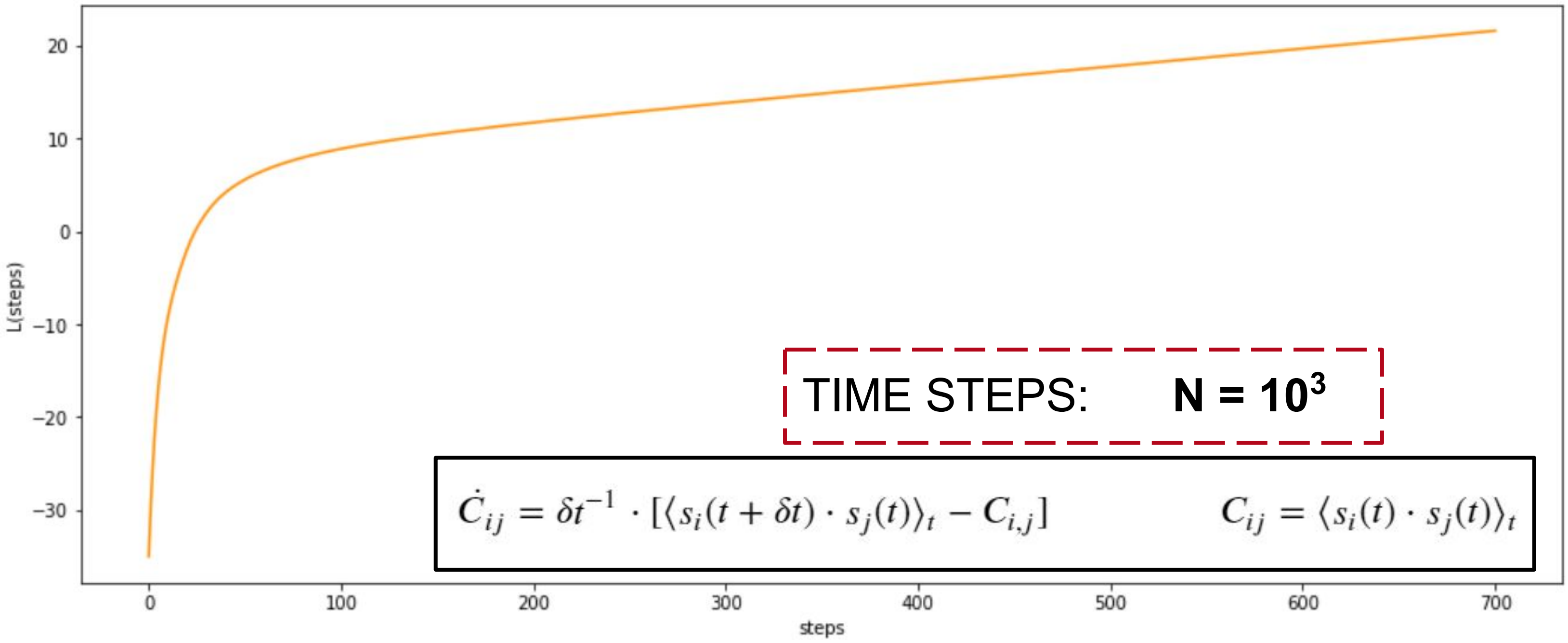


10 neurons: external field h



Convergence of the algorithms

GA convergence profile



TIME STEPS: **N = 10^3**

$$\dot{C}_{ij} = \delta t^{-1} \cdot [\langle s_i(t+\delta t) \cdot s_j(t) \rangle_t - C_{i,j}] \qquad C_{ij} = \langle s_i(t) \cdot s_j(t) \rangle_t$$

Optimization

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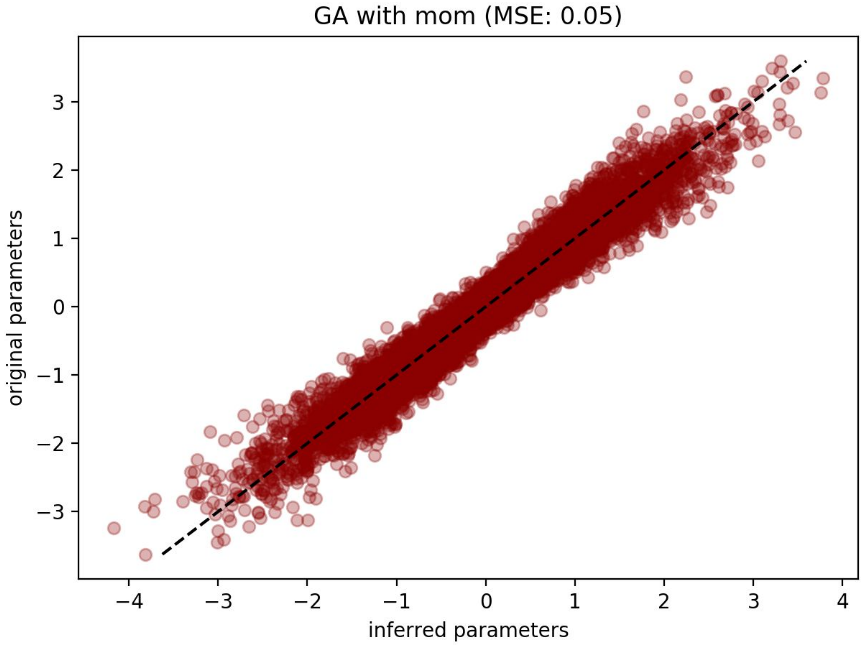
Numpy matrices

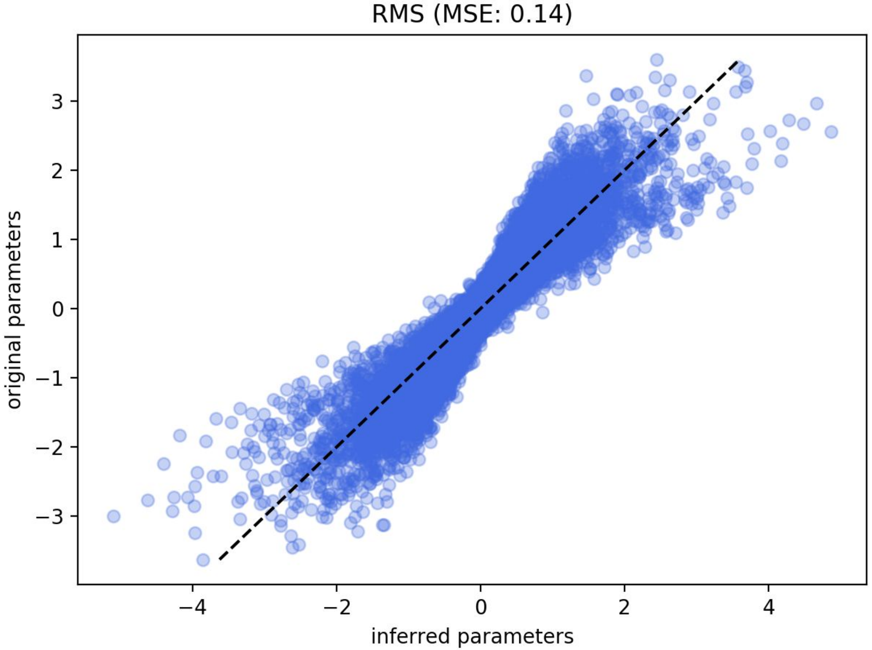
```
def grad_L(p,S,C,C_dot,gamma):
    N_t = S.shape[1]
    N = S.shape[0]

    # p are the params, S is the spins history
    J = p[:N*N].reshape((N,N))      # NxN matrix
    np.fill_diagonal(J,0)           #removing the diagonal
    h = p[N*N:]                     # N vector

    #grad wrt h
    H = (h + np.dot(J,S).T).T
    G_1 = np.mean(S-np.tanh(H),axis=1)

    #grad wrt J
    G_0 = C_dot/gamma + C
    G_0 -= (np.dot(np.tanh(H),S.T))/N_t

    #new
    np.fill_diagonal(G_0,0)
    return np.concatenate((G_0.flatten(),G_1))
```

Dask parallelization on Cloud Veneto to compute the gradient faster

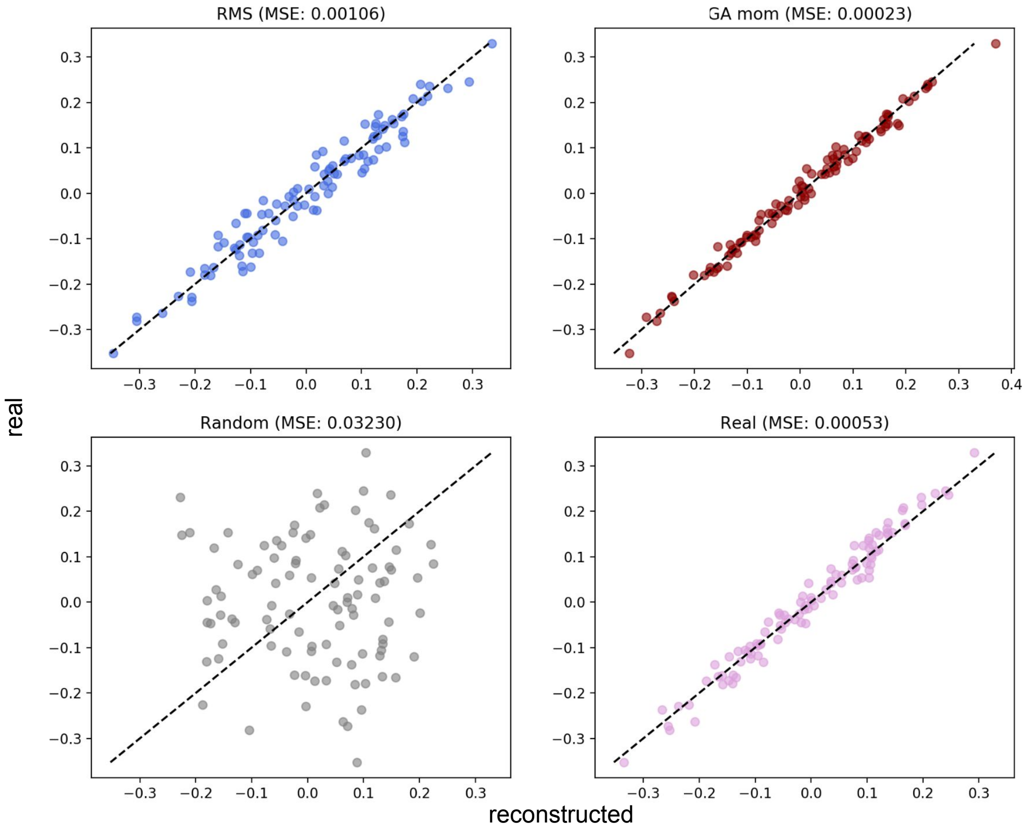
```
def parralel_grad(p,S_split,C,C_dot,gamma):
    ris = []
    for S in S_split:
        ris.append(delayed(grad_L)(p,S,C,C_dot,gamma))
    grad = delayed(sum)(ris).compute()/len(S_split)
    return grad
```

100 neurons: inference

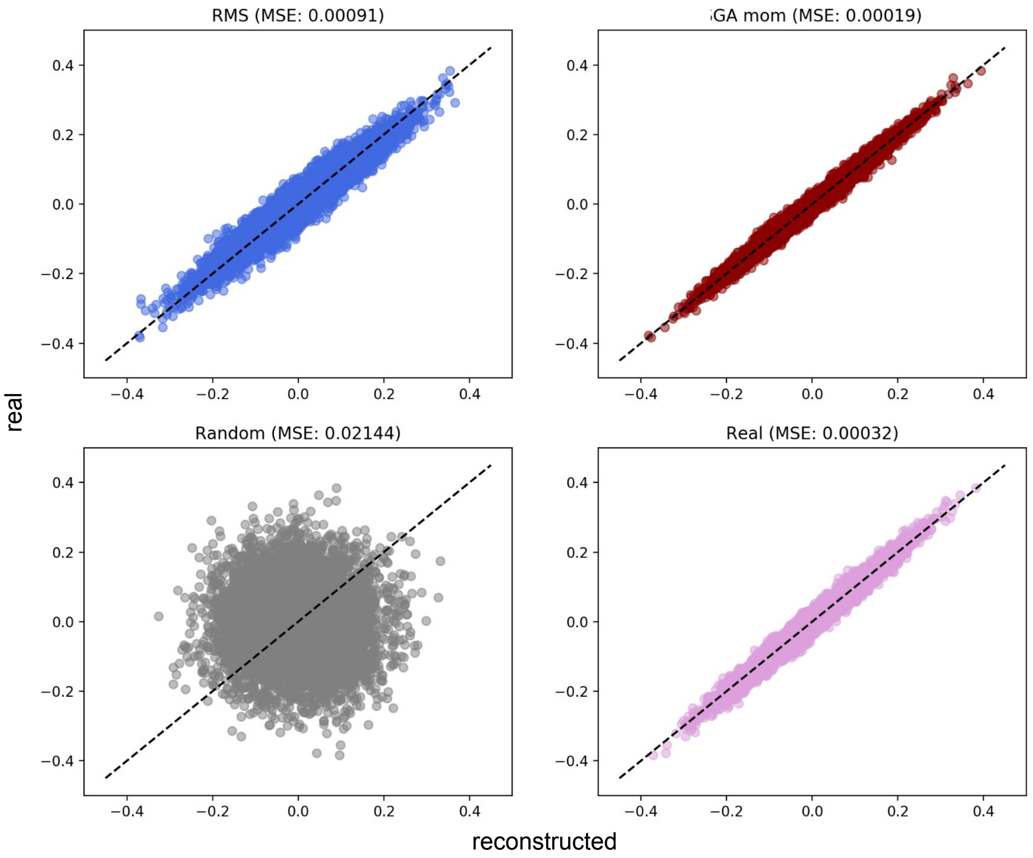




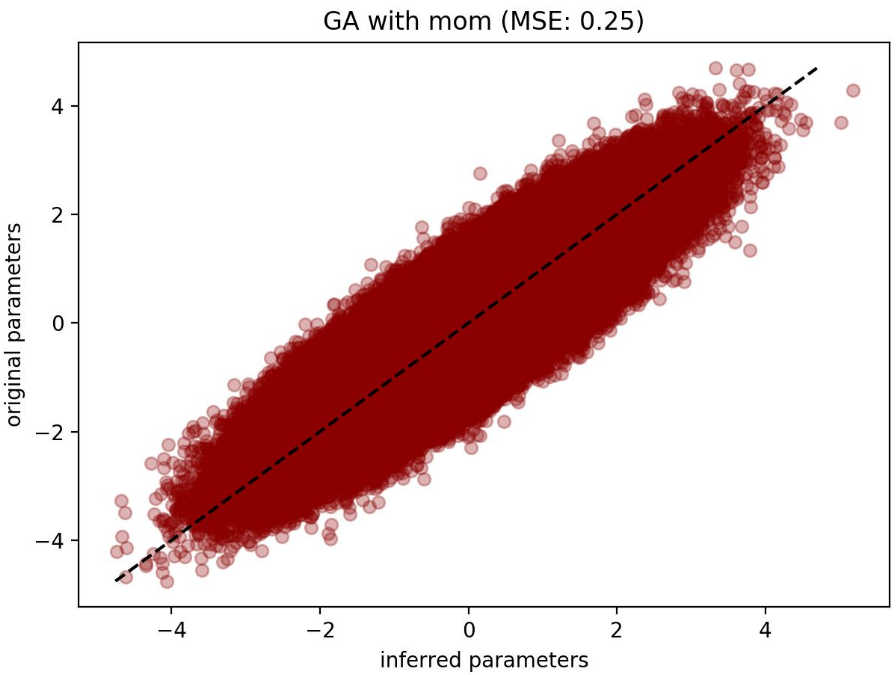
100 neurons: average magnetization per site

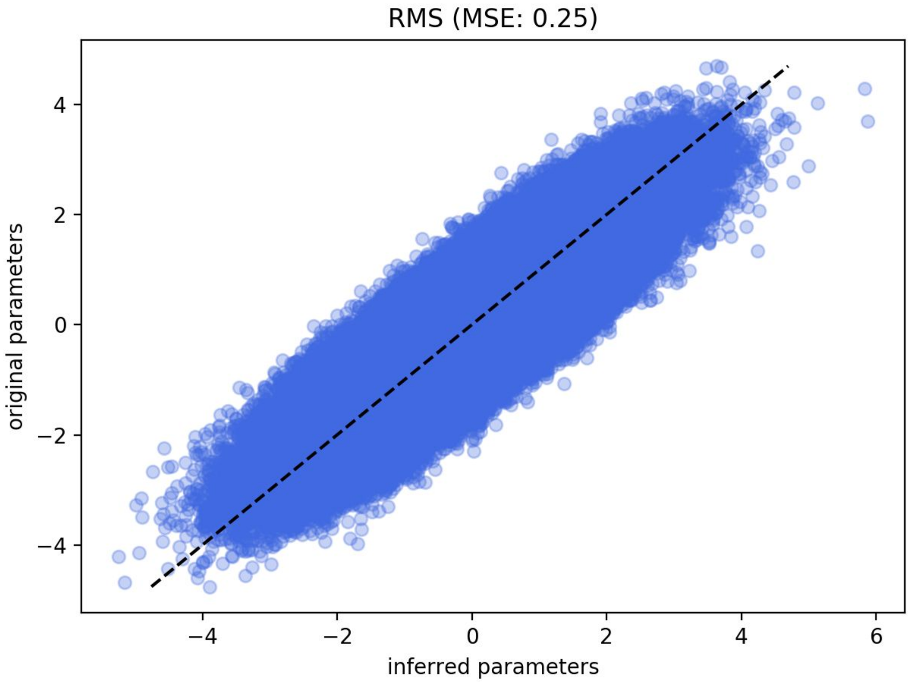


100 neurons: correlation matrix

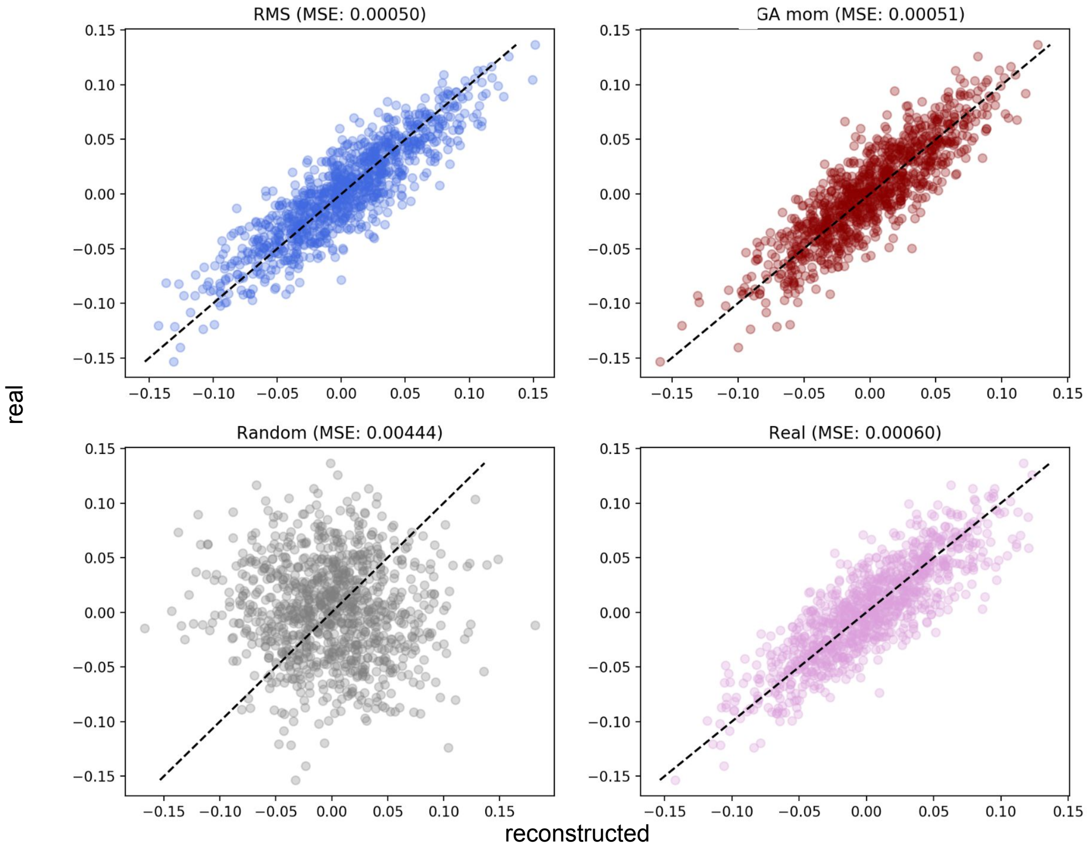


1000 neurons: inference

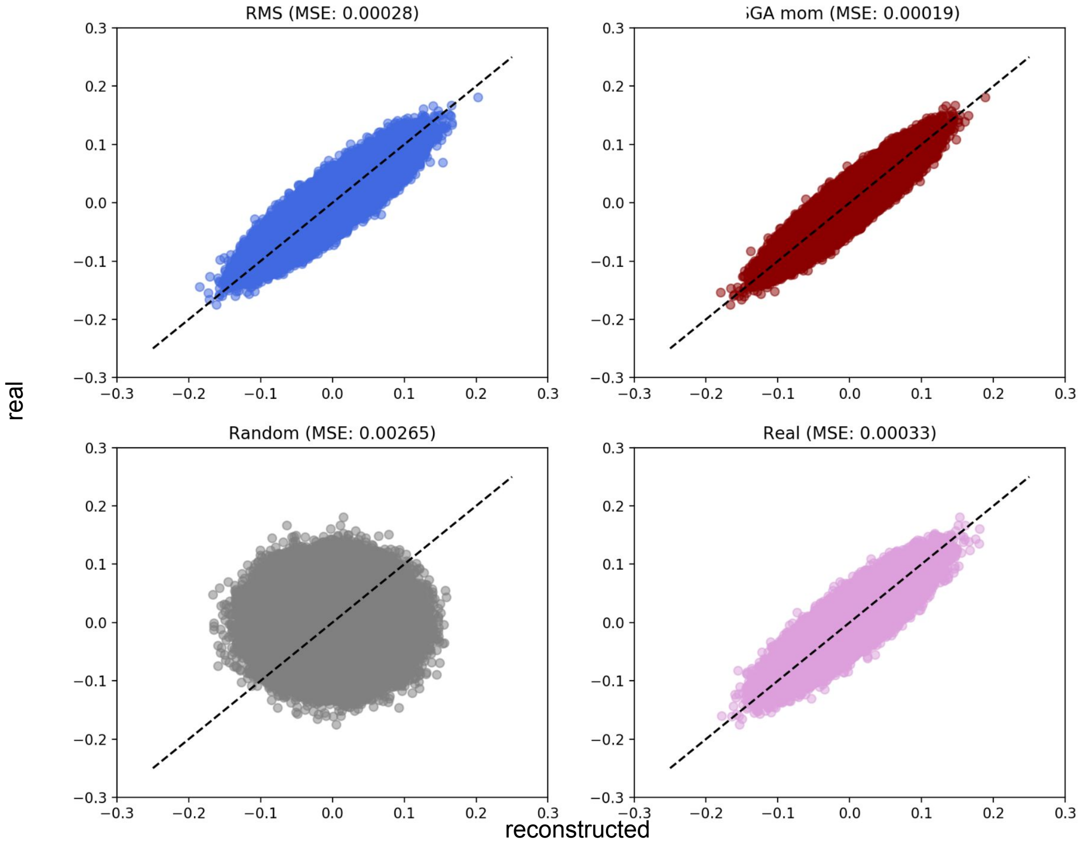




1000 neurons: average magnetization per site

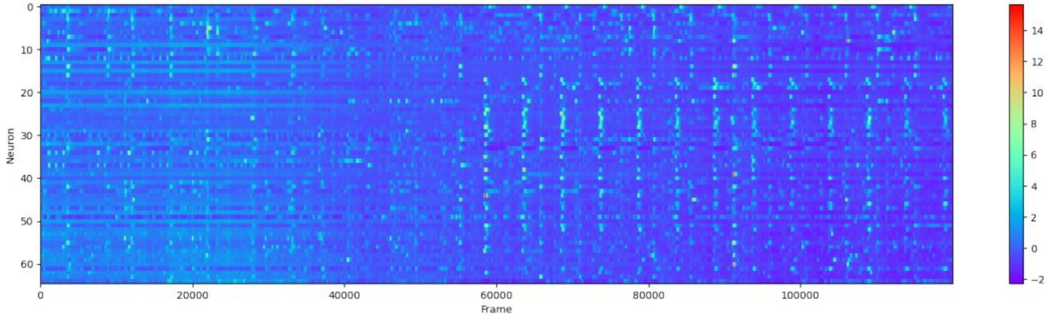


1000 neurons: correlation matrix

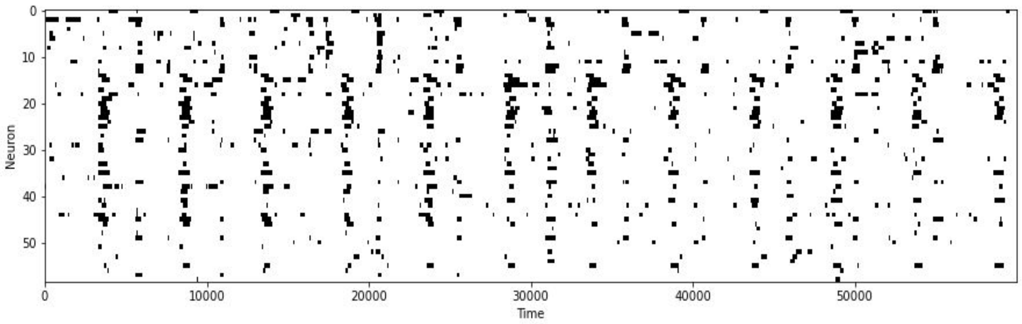


Real Dataset

from Raw Dataset: [3]

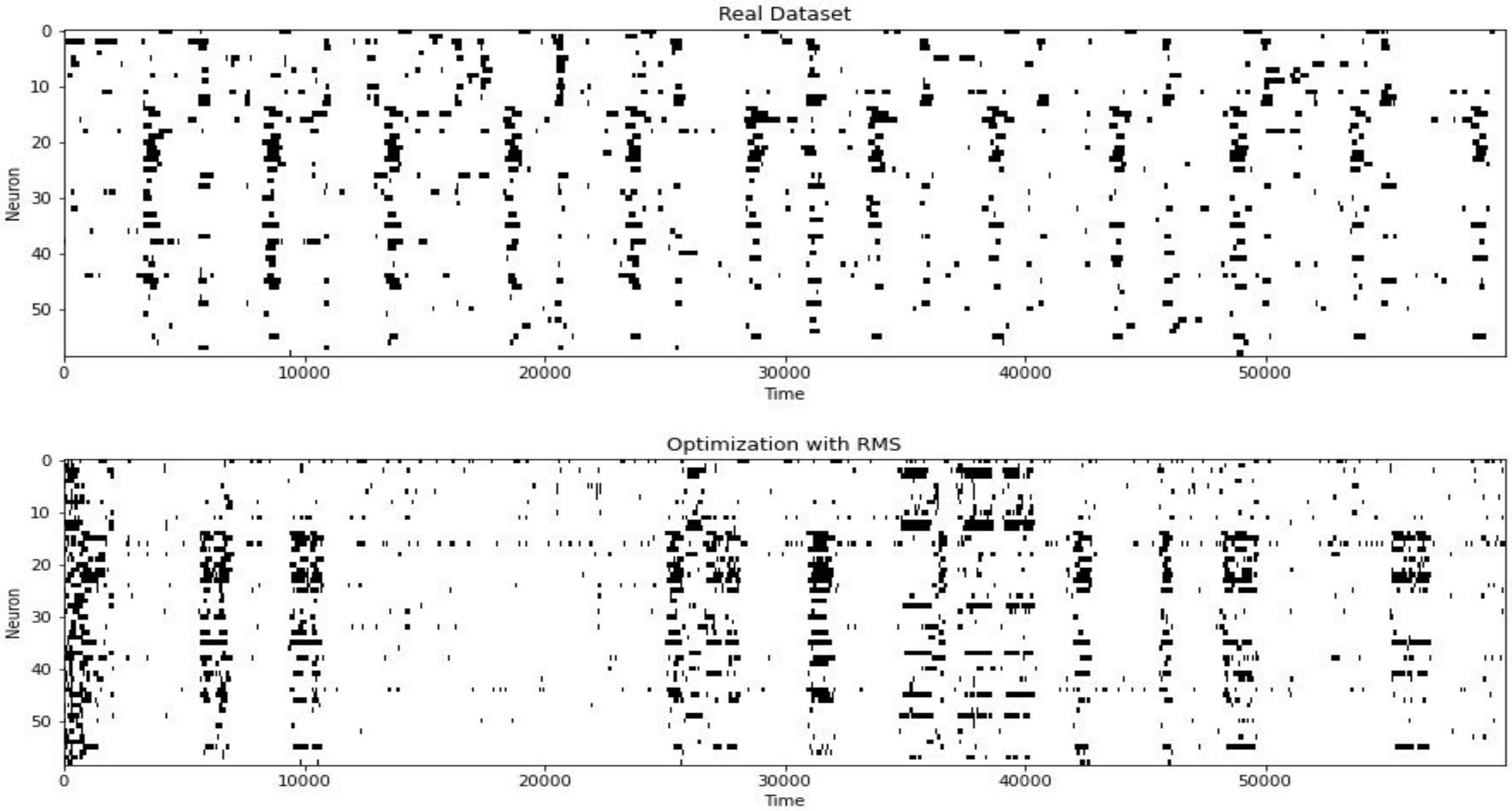


to Pre-Processed Dataset:



➜ discretizing data
➜ removing the first half of the frames without stimuli

Real Dataset: reconstructed dynamic



Real Dataset: reconstructed dynamic

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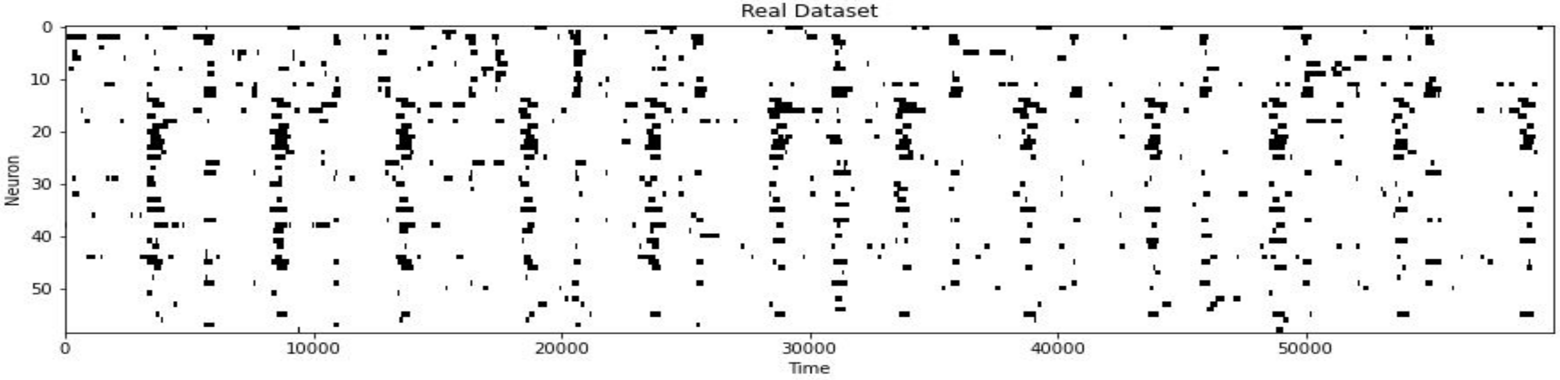
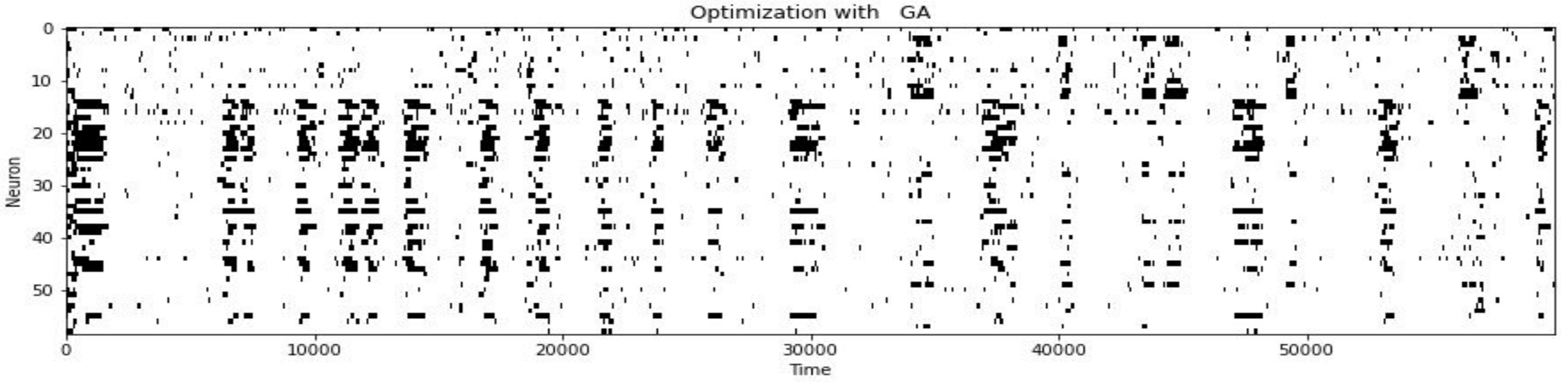
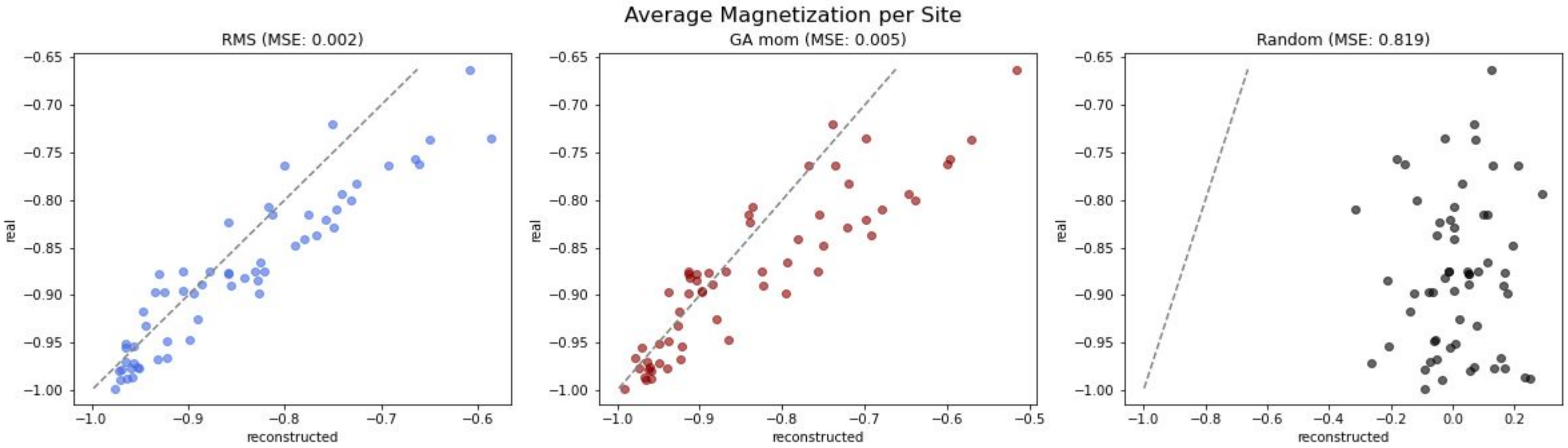
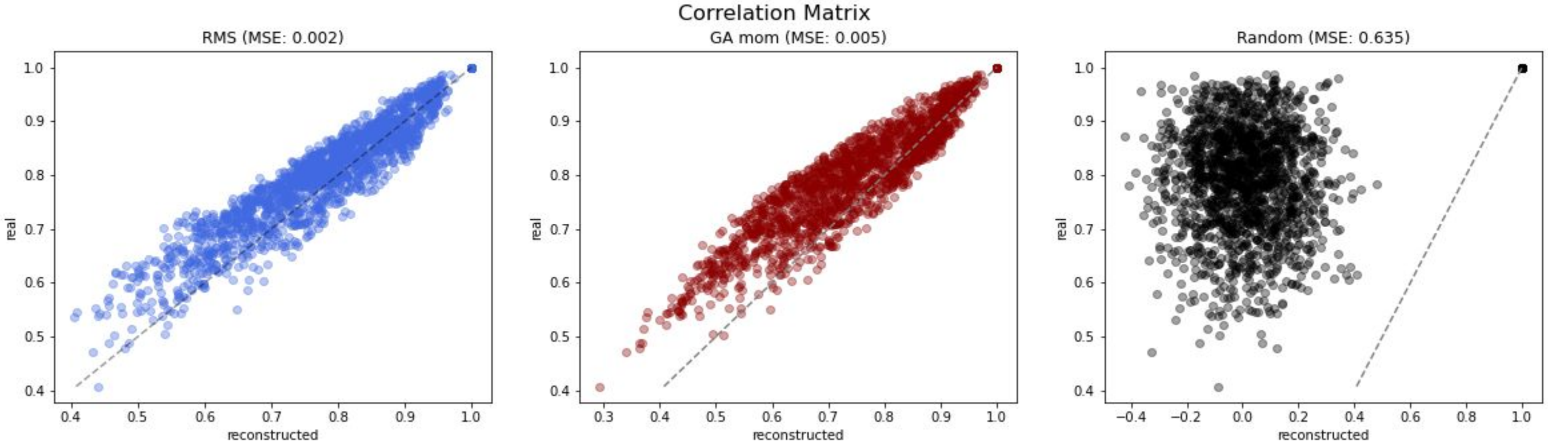




Average Magnetization per site



Correlation Matrix



Conclusions

The reconstruction model works well on synthetic dataset, while on the real dataset the performances get a little worse.

We have found that to work properly, the model needs a big number of time steps ($>10^5$)
➔ a future development is to use real datasets with high number of neurons and time steps

↓

experimental problem:
it's not possible yet to take measurements from an high number of neurons for a long time ➜ trade-off between the two

Possible solution: repeat more times and concatenate together the same timeserie.
But in real life neurons tend to adapt to visual stimuli ➜ this trend should be taken into account on the couplings that now are constant over the time.

Thank you for you attention

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References

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[1] Hong-Li Zeng, Erik Aurell, Mikko Alava, and Hamed Mahmoudi.
Network inference using asynchronously updated kinetic Ising model
Phys. Rev. E **83**, 041135 – Published 29 April 2011

[2] Hong-Li Zeng, Mikko Alava, Erik Aurell, John Hertz, and Yasser Roudi.
Maximum Likelihood Reconstruction for Ising Models with Asynchronous Updates
Phys. Rev. Lett. **110**, 210601 – Published 20 May 2013

[3] Notes provided by the group LIPh.